THE UNIVERSITY OF PRETORIA

FIRST SEMESTER, 2011
Campus: Hatfield

PHYSICS 255

Modern Physics
Exam
Total: 80

(Time allowed: THREE hours)

Internal Examiner: M. van den Worm

External Examiner: Q. Odendaal

NOTE:
- This paper consists of 6 pages.
- Answer all the questions in the answer book.
- A formula sheet is attached at the back of this question paper.
- Use appropriate units, this means that you should use eV and nm when dealing with atomic physics.
- Do not turn the page, wait for instructions.

SECTION A

Special Relativity

1. Your starship passes Earth with a relative speed of 0.9990c. After traveling 10.0 years (your time), you stop at lookout post LP13, turn, and then travel back to Earth with the same relative speed. The trip back takes another 10 years (your time). How long does the round trip take according to measurements made on Earth? (Neglect effects due to accelerations involved with stopping, turning and getting up to speed.) (4 marks)

2. An Earth starship has been sent to check an Earth outpost on the planet P1407, whose moon houses a battle group of the often hostile Reptulians. As the ship follows a straight-line course first past the planet and then past the moon, it detects a high energy microwave burst at the Reptulian moon base and then, 1,10s later, an explosion at the Earth outpost, which is 4.00×10^8 m from the Reptulian base as measured from the ship's reference frame. The Reptulians have obviously attacked the Earth outpost, and so the starship begins to prepare for a confrontation with them. Let subscrips e and b denote the explosion and burst, respectively. (Hint: $\Delta x = x_e - x_b$ and $\Delta t = t_e - t_b$)

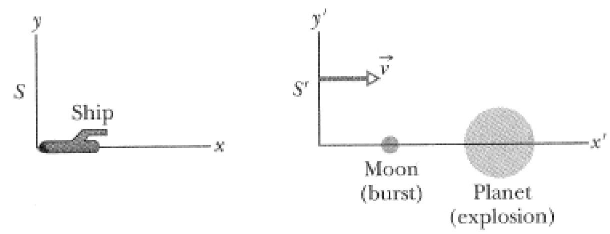

Figure 1: A planet and its moonin reference frame S' move rightward with speed v relative to a starship in reference frame S

(a) The speed of the ship relative to the planet and its moon is 0.980c. What are the distance and time interval between the burst and the explosion as measured in the planet-moon inertial frame (and thus according to the occupants of the stations)?

(b) What is the meaning of the minus sign in the value for $\Delta t'$?

(c) Did the burst cause the explosion, or vice versa? (Remember the postulates of relativity)

(8 marks)

3. What is the total energy E of a 2.53 MeV electron? (3 marks)

4. Prove that if a mass m, acted on by a total force $\overline{F}$, moves a small distance $d\overline{r}$, the change in its energy, dE, equals the work done by $\overline{F}$

$$dE = \overline{F} \cdot d\overline{r} = dW$$

(4 marks)

SECTION B

Quantization

5. (a) Derive the formula for Compton scattering and give the final result in terms of the change in wavelength, $\Delta\lambda$. Fully explain all steps in your derivation.

(b) Through what angle must a 200 keV photon be scattered by a free electron so that the photon loses 10 percent of its energy?

(8 marks)

6. (a) If the work function for a certain metal is 1.8 eV, what is the stopping potential for electrons ejected from the metal when light of wavelength 400 nm shines on the metal?

(b) What is the maximum speed of the ejected electrons?

(6 marks)

7. Consider an electron orbiting around a proton in a circular path with no other external forces present, except that due to the interaction between the electron and the proton.

(a) Show that $mv^2 = \frac{ke^2}{r^2}$.

(b) Show that we can write the total energy of the electron as $E = -\frac{1}{2}\frac{ke^2}{r}$.

(c) Given the fact that angular momenta is quantized in integer multiples of $\hbar$, show that the values of the allowed radii are quantized by $r = n^2 a_B$ where $a_B = \frac{\hbar^2}{ke^2 m}$.

(d) Show that the Bohr model finds the same form as that of the Rydberg formula for the energies of the emitted photons $E_\gamma = hcR\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$ where $R = \frac{ke^2}{2a_B hc}$.

(10 marks)

SECTION C

The Schrödinger Equation

8. Consider an electron trapped in a potential well described by

$$U(x,y) = \begin{cases} 0, & \text{for } 0 \le y \le a \text{ and } 0 \le x \le 2a \\ \infty, & \text{otherwise} \end{cases}$$

Solve the Schrödinger equation for the above potential and determine the wave function and show that the energy can be written an

$$E = E_{n_x,n_y} = \frac{\hbar^2\pi^2}{2Ma^2}\left(\frac{n_x^2}{4} + n_y^2\right)$$

(7 marks)

9. Given that we can write the Laplacian in polar coordinates as

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2}.$$

Set up the differential equations that appear after separating the variables and solve the angular part of Schrödinger equation for the two dimensional central force problem. Comment on the boundary conditions and the effects thereof. (7 marks)

10. (a) The 1s radial wave function of the hydrogen atom is given by

$$R_{1s}(r) = A\exp\left(\frac{-r}{a_B}\right)$$

Show that the amplitude is found to be

$$A = \frac{1}{\sqrt{\pi a_B^3}}$$

(b) Where are we most likely to find en electron in the 1s state of the hydrogen atom?

(7 marks)

SECTION D

Atomic & Nuclear Physics

11. A helium atom is in an energy level with one electron occupying an s state and the other an f state. The two electron spins are antiparallel so that the spin magnetic moments cancel. The atom is placed in a magnetic field $B = 0.8$T.

(a) Sketch the resulting splitting of the original energy level

(b) What is the energy difference between adjacent levels of the resulting multiplet?

(4 marks)

12. State the Pauli exclusion principle (2 marks)

13. 30 g of carbon from the wood from a prehistoric dwelling is ejecting 200 electrons per minute, about how old is the dwelling? (4 marks)

14. (a) Why do isolated neutrons β^--decay as follows

$$n \rightarrow p + e^- + \overline{\nu}$$

(b) Taking into account the Pauli Principle, the Symmetry Effect and IPA draw a diagram and explain why large nuclei require more neutrons to be stable.

(6 marks)

-THE END-

Formula Sheet

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$E^2 = (pc)^2 + (mc^2)^2$$
$$\mu_B = \frac{e\hbar}{2m_e}$$
$$\Delta E = m\mu_B B$$
$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{mc}\,(1-\cos\theta)$$

$$x' = \gamma\,(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$\hbar c = 197 \text{ eV} \cdot \text{nm}$$
$$A_N = 6.022 \times 10^{23} \text{mol}^{-1}$$
$$ke^2 = 1.44 \text{ eV} \cdot \text{nm}$$
$$E_R = 13.6 \text{ eV}$$
$$m_p = 938.3 \text{ MeV}/c^2$$
$$m_n = 939.6 \text{ MeV}/c^2$$
$$m_e = 0.511 \text{ MeV}/c^2$$