THE UNIVERSITY OF PRETORIA

FIRST SEMESTER, 2011
Campus: Hatfield

PHYSICS

Modern Physics
Total: 70

(Time allowed: TWO AND A HALF hours)

NOTE: Answer all the questions. At the end of the paper you will find a survey about this module which I would like to you complete, you will receive 3 free marks for completing the survey. Please fill is this survey on this paper and tear it off when finished.

SECTION A

Quantization of Atomic Energy Levels

1. Consider an electron orbiting around a proton in a circular path with no other forces present.

(a) Show that $mv^2 = \frac{ke^2}{r^2}$.

(b) Show that we can write the total energy of the electron as $E = \frac{-1}{2}\frac{ke^2}{r}$.

(c) Show that angular momentum is quantized; $L = n\hbar$, with $n \in \mathbb{Z}$.

(d) Show that the values of the radii are quantized $r = n^2 a_B$ where $a_B = \frac{\hbar^2}{ke^2 m}$.

(e) Show that the Bohr model finds the same form as that of the Rydberg formula for the energies of the emitted photons $E_\gamma = hcR\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$ where $R = \frac{ke^2}{2a_B hc}$.

(10 marks)

2. The negative muon is a subatomic partivle with the same charge as the electron but a mass what is about 207 times greater. A muon can be captured by a proton to form a "muonic hydrogen atom", with energy and radius given by the Bohr model, except that m_e must be replaced by m_μ. (In this problem treat the proton as fixed.)

(a) What are the radius and energy of the first Borh orbit in a muonic hydrogen atom?

(b) What is the wavelength of the Lyman α line in muonic hydrogen?

(c) What sort of electromagnetic radiation is this?

(6 marks)

SECTION B

Matter Waves

3. According to the Born interpretation of quantum mechanics, what is a matter wave? (2 marks)

4. According to Fourier any periodic function $F(x)$ can be expanded in terms of sines and cosines. If the function happens to be even, $F(x) = F(-x)$, only cosines are needed and the expansion has the form

$$F(x) = \sum_{n=0}^{\infty} A_n \cos\left(\frac{2n\pi x}{\lambda}\right)$$

where λ is the period (or wavelength) of the function.

(a) Prove that

$$A_0 = \frac{1}{\lambda}\int_0^{\lambda} F(x)dx.$$

(b) Prove that for $m > 0$,

$$A_m = \frac{2}{\lambda}\int_0^{\lambda} F(x)\cos\left(\frac{2m\pi x}{\lambda}\right)$$

where we have labeled the coefficient as A_m for reasons that will become clear in your proof.

(7 marks)

5. An unusually long-lived unstable atomic state has a lifetime of 1 ms.

(a) What is the minimum uncertainty in its energy?

(b) Assuming that the photon emitted when this state decays is visible ($\lambda \approx 550$ nm), what are the uncertainty and fractional uncertainty in its wavelength?

(5 marks)

SECTION C

The Schrodinger Equation in One Dimension

6. Without using the Schrodinger equation, show that the energies of a quantum particle inside a one dimensional infinite potential well is quantized by

$$E_n = n^2 \frac{\pi^2\hbar^2}{2ma^2}.$$

(4 marks)

7. Show that you get the same result as above when using the Schrodinger equation. (4 marks)

8. Consider a particle in the ground state of a rigid box of length a.

(a) Find the probability density $|\psi|^2$.

(b) What would be the average result if the position of a particle in the ground state were measured many times?

(6 marks)

9. Draw the first three energy levels and wave function of the simple harmonic oscillator potential. (3 marks)

10. The $n = 1$ wave function for the simple harmonic oscillator potential is given by

$$\psi(x) = A_1 \frac{x}{b} e^{-\frac{x^2}{2b^2}}$$

where $b = \sqrt{\frac{\hbar}{m\omega_c}}$. Verify that this wave function does indeed satisfy the Schrodinger equation with the harmonic oscillator potential and energy. (5 marks)

11. State the time dependent Schrodinger equation (2 marks)

SECTION D

The 3D Schrodinger Equation

12. (a) Using two different methods, show that when solving the Schrodinger equation for the 2D infinite potential well with side lengths a, you find that the energy is given by

$$E_{n_x,n_y} = \frac{\hbar^2\pi^2}{2Ma^2}\left(n_x^2 + n_y^2\right)$$

(b) Determine the aplitude of the ground state wave function.

(13 marks)

-THE END-

SURVEY

Name: ....

Student No: ....

1. If you had to give this module a rating out of 10, what would it be?

2. How do you think this module can be made better for both students and lectures?

3. I am planning to present some popular science lectures for you, I would like an indication of certain topics of interest. We might have a discussion about the current happenings with the LHC, some string theory, GUT, or anything you would like to know about. Please state the topic of interest below.