PHY 255
Modern Physics - Semester Test 1
Total: 80

19 April 2011

Special Relativity 35

1. Derive the the formula for time dilation.

   (a) Draw two diagrams, one depicting the stationary and the other the moving reference frame. Clearly indicate the direction of motion and the path travelled by the light ray. Assume the height of the "cabin" to be h and the moving frame to have speed v.

   (b) What will $\Delta t'$ be? (The time for the beam of light to reach the mirror and be sent back to the detector/sender.)

   (c) Show that Δt can be written in terms of $\Delta t'$ as

   $$\Delta t = \gamma \Delta t'$$

   where $\gamma = \frac{1}{\sqrt{1+\beta^2}}$. (Draw a velocity addition triangle to find the correct answer.)

2. The premise of the Planets of the Apes movies and book is that hibernating astronauts travel far into Earth's future, to a time when human civilization has been replaced by an ape civilization. Considering only special relativity, determine how far into Earth's future the astronauts would travel if they slept for 120 years while traveling relative to Earth with a speed of 0.9990c, first outward from Earth and then back again.

3. In the Mickelson-Morley experiment we have the following diagram which illustrates the concepts behind the experiment:

   (a) Show that the total time for the round trip on leg 1 can be written as

   $$t_1 = \frac{2l}{c}\left(1+\beta^2\right)$$

   (Hint: You will have to make use of the binomial expansion)

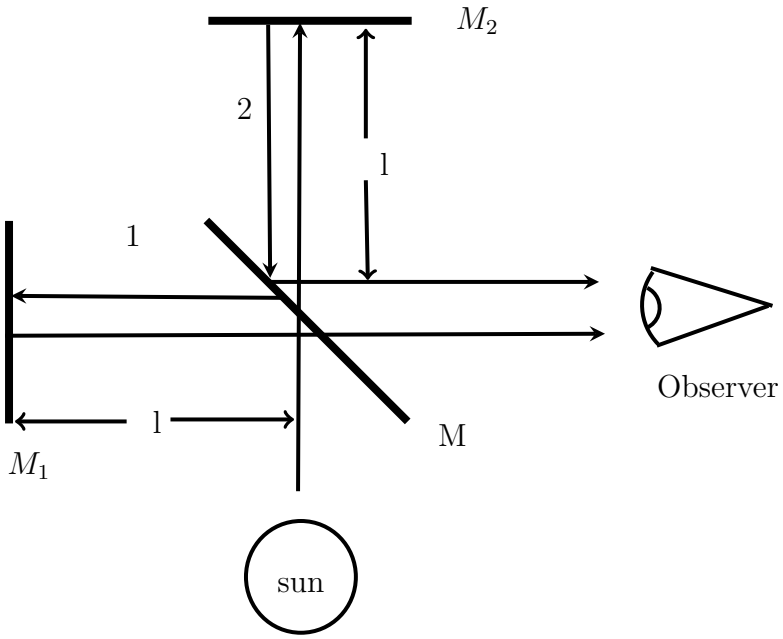

Figure 1: Schematic diagram of the Michelson interferometer.

(b) Show that the total time for the round trip on leg 2 can be written as

$$t_2 = \frac{2l}{c}\left(1 + \frac{1}{2}\beta^2\right)$$

(c) Calculate the difference in time Δt and comment on the answer. (When will we observe constructive and destructive interference?)

4. State the postulates of relativity.

5. Derive the equations for the Lorentz transformations and write these equations in the widely used matrix notation. In your derivation, you should consider a boost along the x-axis of a train cabin. The moving reference frame of the cabin can be denoted by S' and the stationary frame by S.

(a) Draw a diagram which you will use to determine the equations for Lorentz transformations

(b) Using length contraction ($l = \frac{l'}{\gamma}$) show that we can write

$$\begin{aligned} x' &= \gamma\,(x - vt) \\ x &= \gamma\,(x' + vt) \end{aligned}$$

(c) Show that $t' = \gamma\left(t - \frac{vx}{c^2}\right)$

(d) Show that we can write

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

6. A relativistic snake of proper length 100cm is moving at speed $v = 0.6c$ to the right across a table. A boy, whishing to tease the snake, holds two hatchets 100cm apart and plans to bounce them simultaneously on the table so that the left hatchet lands immediately behind the snake's tail. Denote the rest frame of the snake as the primed reference frame.

   (a) By how far will the second hatchet miss the snake?

   (b) The two hatchets do not strike the table simultaneously in the snake's frame. Calculate the difference in time between the strikes of the hatchets in the snake's rest frame of reference.

   (c) Show that the snake will not be chopped by the boy, i.e. the distance between the hatchets in the snake's rest frame is longer that the proper length of the snake.

Relativistic Mechanics [12]

7. Given that

$$\begin{aligned} \overline{p} &= \gamma m \overline{u} \\ E &= \gamma m c^2, \end{aligned}$$

   derive the "Pythagorean Relation" $E^2 = (pc)^2 + (mc^2)^2$.

8. A particle of unknown mass M decays into two particles of known masses $m_1 = 0.5\text{GeV}/c^2$ and $m_2 = 1\text{Gev}/c^2$, whose momenta are measured to be $\overline{p}_1 = 2\text{GeV}/c$ along the positive y-axis and $\overline{p}_2 = 1.5\text{GeV}/c$ along the positive x-axis ($1GeV = 10^9\text{eV}$). Find the unknown mass M and its speed.

Atoms [8]

9. Derive a formula which determines the average distance travelled by a gas molecule in some container. Assume the molecule collides with N other gas molecules, the total displacement of the particle is

$$\overline{D} = \sum_{i=1}^{N} \overline{d}_i.$$

   Using the root-mean-squared average show that

$$D_{rms} = \sqrt{N}\lambda.$$

   Where λ denoted the mean free path.

10. An aimless physics student performs a random walk by taking steps of one meter at a rate of one step per second each second. These steps are taken in completely random directions. How long will it take the student to have travelled one kilometer away from the starting position?

Quantization of Light [25]

11. (a) In the photoelectric effect we observe two phenomena which cannot be explained be classical physics, state these phenomena.

    (b) How can these two phenomena be explained using the ideas of quantization of light?

12. In a photoelectric experiment using a sodium surface, you find a stopping potential of 1.85 V for a wavelength of 300nm and a stopping potential of 0.820 V for a wavelength of 400nm.

    (a) Find a value for Planck's constant using these values.

    (b) Find the work function ϕ for sodium.

    (c) Find the cutoff wavelength λ_0 for sodium.

13. (a) How are X-rays created?

    (b) What are the typical energies and wavelenghts we expect to see when dealing with X-rays?

    (c) Derive Bragg's Law ($2d\sin\theta = n\lambda$).

14. Derive the formula for Compton scattering and give the final result in terms of the change in wavelength, $\Delta\lambda$.

15. A 100W beam of light shines for 1000 seconds on a 1g black object initially at rest in a frictionless environment. The object absorbs all the light of the beam.

    (a) Calculate the total energy and momentum of the photons absorbed by the blackbody.

    (b) Find the body's final velocity.

    (c) Calculate the body's final kinetic energy. How can this be less than the original energy of the photons?