THE UNIVERSITY OF PRETORIA

FIRST SEMESTER, 2011
Campus: Hatfield

PHYSICS

Modern Physics
Sick Test
Total: 70

(Time allowed: TWO AND A HALF hours)

SECTION A

Special Relativity

1. State the postulates of relativity (2 marks)

2. Given that

$$\begin{aligned} \overline{p} &= \gamma m \overline{u} \\ E &= \gamma m c^2, \end{aligned}$$

derive the "Pythagorean Relation" $E^2 = (pc)^2 + \left(mc^2\right)^2$. (4 marks)

SECTION B

Quantization of Light

3. In a photoelectric experiment using a sodium surface, you find a stopping potential of 1.85 V for a wavelength of 300nm and a stopping potential of 0.820 V for a wavelength of 400nm.

(a) Find a value for Planck's constant using these values.

(b) Find the work function ϕ for sodium.

(c) Find the cutoff wavelength λ_0 for sodium.

(7 marks)

SECTION C

Schrodinger Equation

4. Consider the following function describing the potential energy of a particle confined to an infinite potential well

$$U(x,y) = \begin{cases} 0, & \text{if } 0 \le x \le a \text{ and } 0 \le y \le b \\ \infty, & \text{otherwise} \end{cases}$$

(a) Show that the energy can be written as

$$E_{n_x,n_y} = \frac{\hbar^2\pi^2}{2M}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2}\right)$$

(b) Determine the aplitude of the ground state wave function.

(12 marks)

5. (a) Draw a diagram showing where we find the values of

$$\begin{aligned} x = r\cos\theta \quad &, \quad y = r\sin\theta \\ r^2 = x^2 + y^2 \quad &, \quad \tan\theta = \frac{y}{x} \end{aligned}$$

when dealing with polar coordinates.

(b) Using the chain rule we can write

$$\frac{\partial}{\partial x} = \frac{\partial r}{\partial x}\frac{\partial}{\partial r} + \frac{\partial \theta}{\partial x}\frac{\partial}{\partial \theta}$$

and similarly for $\frac{\partial}{\partial y}$. Show that we can write these two expressions as

$$\begin{aligned} \frac{\partial}{\partial x} &= \cos\theta\frac{\partial}{\partial r} - \frac{1}{r}\sin\theta\frac{\partial}{\partial \theta} \\ \frac{\partial}{\partial y} &= \sin\theta\frac{\partial}{\partial r} + \frac{1}{r}\cos\theta\frac{\partial}{\partial \theta}. \end{aligned}$$

(c) Using the above result, show that we can write

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial \phi^2}. \tag{1}$$

(14 marks)

6. Given that we can write the Laplacian in polar coordinates as in equation 1, find differential equations which can be used to solve the angular and radial parts of the Schrodinger equation separately. (4 marks)

SECTION D

Hydrogen Atom

7. (a) We seek to find the probability of finding en electron in the spherical shell between radii r andn $r+dr$. Recall that the probability of finding an electron within a small volume is

$$|\psi|^2 dV.$$

Show that the radial probability density can be written as

$$P(r) = 4\pi r^2 |R(r)|.$$

(b) The 1s radial wave function of the hydrogen atom is given by

$$R_{1\mathrm{s}}(r) = Ae^{\frac{-r}{a_B}}$$

Show that the amplitude is found to be

$$A = \frac{1}{\sqrt{\pi a_B^3}}$$

(c) Where are we most likely to find en electron in the 1s state of the Hydrogen atom?

(10 marks)

SECTION E

Electron Spin and Moments

8. State the Pauli Exclusion Principle. (2 marks)

9. Let us consider an electron in the lowest p-state of the Hydrogen atom. Write all the possible values of the quantum numvers n, l, m and m_s (2 marks)

10. (a) If we consider an electron as a point particle traveling around the nuclues we can think of this orbiting charge as a small current loop. We assume the electron is traveling with velocity v and traverses its circular path in time T. Show that the current in this loop is

$$I = \frac{ev}{2\pi r}$$

(b) Recall that we can write the magnitude of the angular magnetic moment of the electron as

$$\mu = IA.$$

Show that we have the following ralationship between angular momentum and the angular magnetic moment

$$\overline{\mu} = -\frac{e}{2m_e}\overline{L}$$

and comment on the negative sign.

(6 marks)

11. Imagine a hydrogen atom in which the electron has no spin (zero spin magnetic moment). The atom is placed in a magnetic field $B = 1.5$ T along the z-axis.

(a) Describe the effect of the B-field on the 1s and 2p states of the hypothetical atom.

(b) Sketch the energy levels for when the B-field is on and off.

(c) When the B-field is on we should observe a splitting of the energy levels. What will be the separation of the frequencies of the observed photons which were produced form the de-excitations of the split energy levels?

(10 marks)

-THE END-