THE UNIVERSITY OF PRETORIA

FIRST SEMESTER, 2011
Campus: Hatfield

PHYSICS 255

Modern Physics
Exam
Total: 70

(Time allowed: THREE hours)

Internal Examiner: M. van den Worm

External Examiner: Q. Odendaal

NOTE:
- This paper consists of 5 pages.
- Answer all the questions in the answer book.
- A FORMULA SHEET is attached at the BACK of this question paper.
- Use appropriate units, this means that you should use eV and nm when dealing with atomic physics.
- Do not turn the page, wait for instructions.

SECTION A - Special Relativity

1. The elementary particle known as the positive kaon (K^+) has, on average, a lifetime of 0.1237 μs when stationary - that is, when the lifetime is measured in the rest frame of the kaon. If a positive kaon has a speed of 0.990c relative to a laboratory reference frame when the kaon is produced, how far can it travel in that frame during its lifetime according to

(a) classical physics and [2]

(b) special relativity? [2]

2. An observer in stationary frame S detects two flaches of light. A big flash occurs at $x_1 = 1200$ m and, 5.00 μs later, a small flash occurs at $x_2 = 480$ m. As detected by the moving observer S', the two flashes occur at a single coordinate x'.

(a) What is the speed parameter of S'? [2]

(b) Is S' moving in the positive or negative direction of the x-axis? [2]

(c) To S', which flash occurs first? [2]

(d) To S', what is the time interval between the flashes? [2]

3. What is the momentum in MeV/c of an electron with a kinetic energy of 2.00 MeV? [2]

SECTION B - Quantization

4. (a) Derive the formula for Compton scattering and give the final result in terms of the change in wavelength, $\Delta\lambda$. Fully explain all steps in your derivation. [4]

(b) Gamma rays of photon energy 0.511 MeV are directed onto an aluminum target and are scattered in various direction by loosely bound electron there.

(i) What is the wavelength of the incident gamma rays? [2]

(ii) What is the wavelength of gamma rays scattered at 90°? [2]

(iii) What is the photon energy of the rays scattered in this direction? [2]

5. (a) What is the *photoelectric effect*? [2]

(b) (i) The smallest amount of energy needed to eject an electron from metallic sodium is 2.28 eV. Does sodium show a photoelectric effect for red light, with $\lambda = 680$ nm? [2]

(ii) What is the cutoff wavelength for photoelectric emission from sodium? [2]

(iii) To what color does that wavelength correspond? (Hint: Visible light has $\lambda \in [400, 700]$ nm with violet on the low side and red on the high side of the spectrum) [2]

6. (a) What is the wavelength of light for the least energetic photon emitte din the Lyman series of the hydrogen atom specrum lines? [2]

(b) What is the wavelength of the series limit for thy Lyman series? [2]

SECTION C - The Schrödinger Equation

7. According to the Born interpretation of Quantum Mechanics, what is the physical interpretation of a matter wave? [2]

8. Consider the following crude approximation of an atom described by the the potential

$$U(x,y,z) = \begin{cases} 0, & \text{for } 0 \le x \le a,\ 0 \le y \le 2a \text{ and } 0 \le z \le 3a \\ \infty, & \text{otherwise} \end{cases}$$

(a) By solving the Schrödinger equation for the above potential show that the wavefunction is given by

$$\psi(x,y,z) = A\sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{2a}\right)\sin\left(\frac{n_z \pi z}{3a}\right),$$

[3]

(b) Show that the amplitude of the wave function is

$$A = \frac{2}{\sqrt{3a^3}}.$$

[3]

(c) Show that the energy of a particle trapped inside this quantum well can be written as

$$E = E_{n_x,n_y} = \frac{\hbar^2\pi^2}{2Ma^2}\left(n_x^2 + \frac{n_y^2}{4} + \frac{n_z^2}{9}\right).$$

[2]

(d) Show that for the ground state wave function we are most likely to find the particle at

$$\langle (x,y,z) \rangle = \left\langle \left(\frac{a}{2}, a, \frac{3a}{2}\right) \right\rangle.$$

[3]

9. (a) Write down the one dimentional time dependant Schrödinger Equation. [2]

(b) Show that for standing wave solutions $\Psi(x,t) = \psi(x)e^{-i\omega t}$ the spatial wave function $\psi(x)$ satisfies the one dimensional time independant Schrödinger equation. [3]

10. Make sketches of the probability density $|\psi(x)|^2$ for the first four energy levels of a particle in a 1D nonrigid box. Draw next to each graph the corresponding graph for the 1D rigid box. [3]

SECTION D - Atomic & Nuclear Physics

11. A helium atom is in an energy level with one electron occupying an s state and the other an f state. The two electron spins are antiparallel so that the spin magnetic moments cancel. The atom is placed in a magnetic field $B = 0.8$T.

(a) Sketch the resulting splitting of the original energy level [2]

(b) What is the energy difference between adjacent levels of the resulting multiplet? [2]

12. State the Pauli exclusion principle [2]

13. A 1 g sample of samarium emits alpha particles at a rate of 120 particles/s. The responsible isotope is ^{147}Sm, whose natual abundance in bulk samarium is 15%.

(a) Show that the number of ^{147}Sm nuclei present in the sample is

$$N = 6.143 \times 10^{20}.$$

[2]

(b) Calculate the falf-life for the decay process. [2]

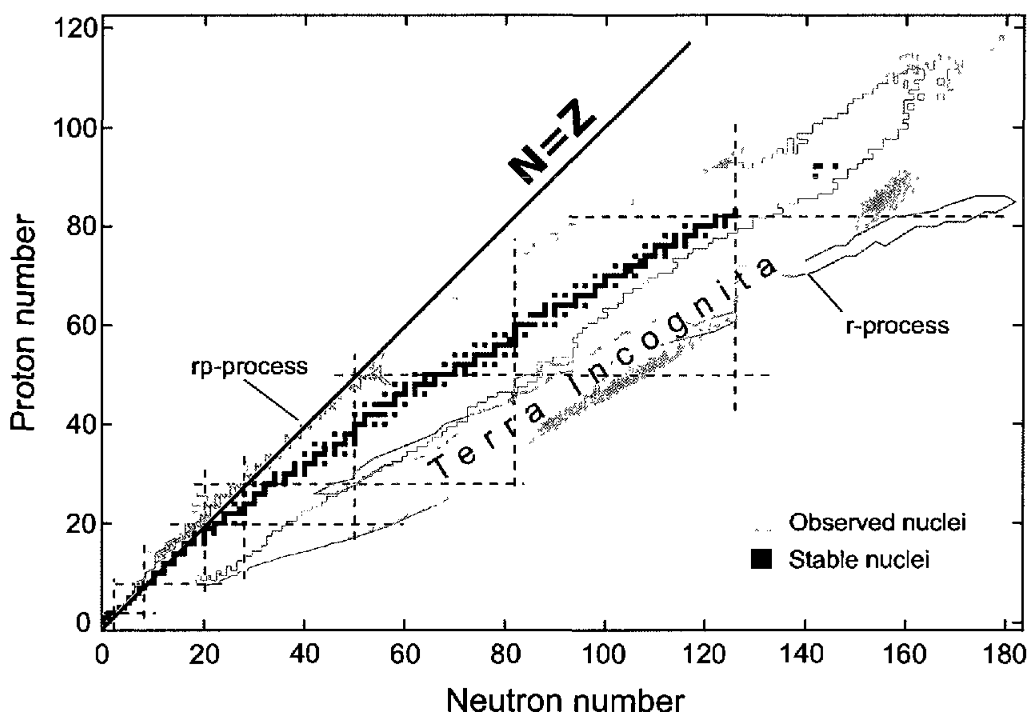

Figure 1: Figure showing the stable nuclei together with the $Z = N$ reference line.

14. Taking into account the Pauli Principle, the Symmetry Effect and IPA draw a diagram and explain why large nuclei require more neutrons to be stable as seen in figure 1. [3]

-THE END-

Formula Sheet

$$
\begin{aligned}
\int \sin^2(ax)dx &= \frac{x}{2} - \frac{1}{4a}\sin(2ax) + C \\
\int x\sin^2(ax)dx &= \frac{x^2}{4} - \frac{x}{4a}\sin(2ax) - \frac{1}{8a^2}\cos(2ax) + C \\
\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\
E^2 &= (pc)^2 + (mc^2)^2 \\
\mu_B &= \frac{e\hbar}{2m_e} \\
\Delta E &= m\mu_B B \\
\Delta\lambda &= \lambda - \lambda_0 = \frac{h}{mc}(1-\cos\theta) \\
V_s e &= K_{\max} = hf - \phi \\
N(t) &= N_0 e^{rt} \\
\tau &= \frac{1}{r} \\
t_{1/2} &= \tau \ln 2
\end{aligned}
$$

$$
\begin{aligned}
x' &= \gamma\,(x - vt) \\
y' &= y \\
z' &= z \\
t' &= \gamma\left(t - \frac{vx}{c^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
\hbar c &= 197 \text{ eV} \cdot \text{nm} \\
hc &= 1240 \text{ eV} \cdot \text{nm} \\
A_N &= 6.022 \times 10^{23} \text{mol}^{-1} \\
ke^2 &= 1.44 \text{ eV} \cdot \text{nm} \\
E_R &= 13.6 \text{ eV} \\
m_p &= 938.3 \text{ MeV}/c^2 \\
m_n &= 939.6 \text{ MeV}/c^2 \\
m_e &= 0.511 \text{ MeV}/c^2 \\
1u &= 1.66 \times 10^{-27} \text{ kg}
\end{aligned}
$$