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THE UNIVERSITY OF PRETORIA

FIRST SEMESTER, 2012
Campus: Hatfield

PHYSICS 255

Modern Physics
Total: 80

(Time allowed: TWO hours)

Internal Examiner: M. van den Worm

External Examiner: Q. Odendaal

NOTE:
- This question paper consists of 8 pages, 5 sections (A - E) and 14 questions.
- Answer all the questions.
- Use correct notation, indicate vectors by $\overline{x} = (a, b, c)$.
- When deriving formulas indicate clearly what you are doing.
- Try and work neat, it is easier to grade a neat paper.
- DO NOT TURN THE PAGE

SECTION A

Relativity

1. Using the figures below derive the formula for time dilation (6 marks)

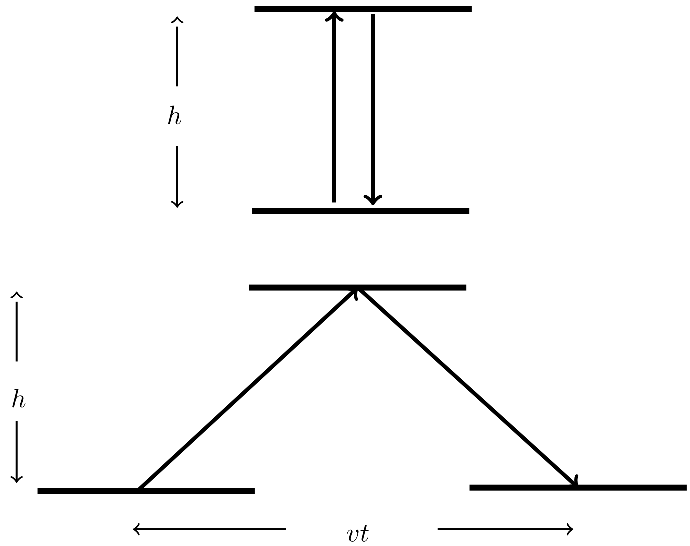


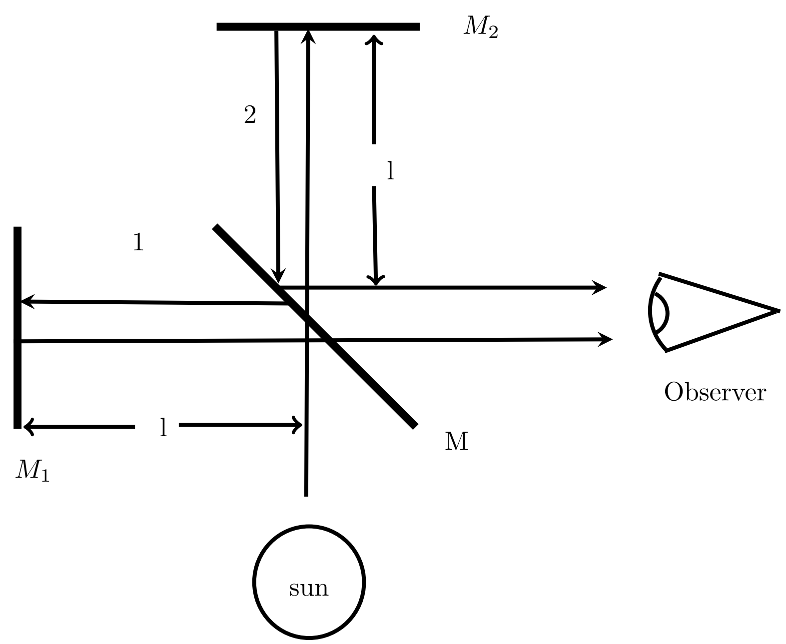
Figure 1: The top figure shows what an observer in the moving reference frame will observe while the bottom figure shows that what an outside observer will observe.

2. Bill and Julie are both now on identical trains. Bill's train is moving to the right with velocity $\frac{\sqrt{3}}{2}c$ with respect to Julie's train. Julie measures her train to be 100 meters long.

(a) How long does Julie measure Bill's train to be?

(b) How long does Bill measure Julie's train to be?

(4 marks)



3. In the Mickelson-Morley experiment we have the diagram above which illustrates the concepts behind the experiment:

(a) Show that the total time for the round trip on leg 1 can be written as

$$t_1 = \frac{2l}{c}\left(1 + \beta^2\right)$$

(Hint: You will have to make use of the binomial expansion)

(b) Show that the total time for the round trip on leg 2 can be written as

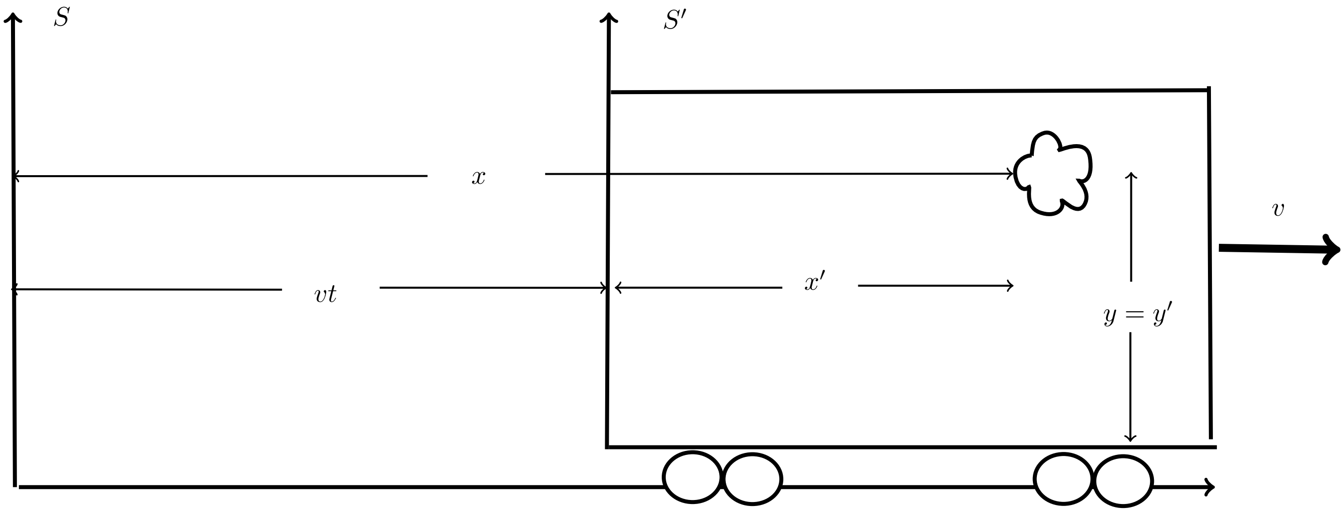
$$t_2 = \frac{2l}{c}\left(1 + \frac{1}{2}\beta^2\right)$$

(c) Calculate the difference in time Δt and comment on the answer.

(d) When will we observe constructive and destructive interference?

(10 marks)

4. Using length contraction, time dilation and the figure below show that



(a)

$$x' = \gamma\,(x - vt)\,, \quad x = \gamma\,(x' + vt) \qquad (1)$$

where primes indices represent the moving reference frame.

(b) Show that $t' = \gamma\left(t - \frac{vx}{c^2}\right)$

(c) Show that we can write

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

(6 marks)

5. A clock moves along the x-axis at a speed of $0.6c$ and reads zero as it passes the origin. What time does the clock read as it passes $x = 180$ m? (4 marks)

SECTION B

Relativistic Mechanics

6. Given that

$$\overline{p} = \gamma m \overline{u}$$
$$E = \gamma m c^2,$$

derive the "Pythagorean Relation" $E^2 = (pc)^2 + \left(mc^2\right)^2$. (6 marks)

7. A particle of unknown mass M decays into two particles of known masses $m_1 = 0.5\mathrm{GeV}/c^2$ and $m_2 = 1\mathrm{Gev}/c^2$, whose momenta are measured to be $\overline{p}_1 = 2\mathrm{GeV}/c$ along the positive y-axis and $\overline{p}_2 = 1.5\mathrm{GeV}/c$ along the positive x-axis ($1GeV = 10^9$eV). Find the unknown mass M and its speed. (4 marks)

SECTION C

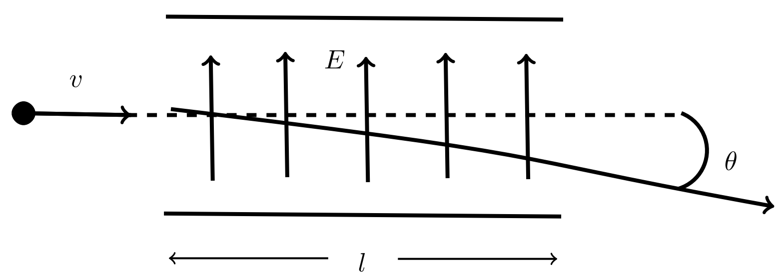
Atoms and the discovery of the electron

8. Draw a diagram depicting the experimental setup used by Thomson to show the existence of the electron and comment of the experiment. Include remarks about the Lorentz force, $\overline{F} = -e(\overline{E} + \overline{v} \times \overline{B})$, ionization, potential difference, charge-to-mass ration, etc. (4 marks)

9. In Thomson's experiment electrons travel with velocity v in the x-direction. They enter a uniform elecric field E, which point in the y-direction and has a total width l. Find the time for an electron to cross the field and the y-component of its velocity when it leaves the field. Hence show that its velocity is deflected through an angle

$$\theta \approx \frac{eEl}{mv^2} \tag{2}$$

provided that θ is small. Assume that the electrons are nonrelativistic, as was the case for Thomson.



(6 marks)

(Hint: Recall the radial definition of an angle, $\theta = \frac{\text{arc length}}{\text{radius}}$)

SECTION D

Quantization of Light

10. (a) What is the photoelectric effect?

(b) If the intensity of the incident light is increased, the number of ejected electrons increases (as one might expect), but the Kinetic energy does not change at all. Explain this in terms of the quantization of eletromagnetic radiation.

(c) If the frequency f of the incident light is reduced below a certain critical frequency f_0 no electrons are ejected however intence the light may be. Explain this in terms of the quantization of electromagnetic radiotion.

(6 marks)

11. The longest wavelength of light that can eject electrons from potassium is $\lambda_0 = 560$ nm.

(a) What is the work function of patassium?

(b) If UV radiation with $\lambda = 300$ nm shines on potassium, what is the stopping potential V_s?

(4 marks)

12. Derive Compton's formula for the change in wavelength of photon scattering of a stationary electron. In other words, derive the equation

$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{mc}\left(1 - \cos\theta\right). \tag{3}$$

Make use of a diagram to aid your derivation. (6 marks)

13. If the maximum kinetic energy given to the electrons in a Compton scattering experiment is 10 keV, what is the wavelength of the incident X-rays? (4 marks)

SECTION E

Quantization of Atomic energy levels

14. Consider an electron orbiting around a proton in a circular path with no other external forces present, except that due to the interaction between the electron and the proton.

(a) Show that $mv^2 = \frac{ke^2}{r^2}$.

(b) Show that we can write the total energy of the electron as $E = -\frac{1}{2}\frac{ke^2}{r}$.

(c) Given the fact that angular momenta is quantized in integer multiples of $\hbar$, show that the values of the allowed radii are quantized by $r = n^2 a_B$ where $a_B = \frac{\hbar^2}{ke^2 m}$.

(d) Show that the Bohr model finds the same form as that of the Rydberg formula for the energies of the emitted photons $E_\gamma = hcR\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$ where $R = \frac{ke^2}{2a_B hc}$.

(10 marks)

-THE END-