THE UNIVERSITY OF PRETORIA

FIRST SEMESTER, 2011
Campus: Hatfield

PHYSICS 255

Modern Physics
Exam
Total: 60

(Time allowed: TWO hours)

Internal Examiner: M. van den Worm

External Examiner: Q. Odendaal

NOTE:
- This paper consists of 6 pages.
- Answer all the questions in the answer book.
- A formula sheet is attached at the back of this question paper.
- Use appropriate units, this means that you should use eV and nm when dealing with atomic physics.
- Do not turn the page, wait for instructions.

SECTION A

Special Relativity

1. State the *Postulates of Relativity.* (2 marks)

2. (a) Using a clear diagram derive the formula for time dilation.

(b) A space traveler takes off from Earth and moves at speed 0.99c toward the star Vega, which is 26 ly distant. How much time will have elepsed by Earth clocks

(i) when the traveler reaches Vega and

(ii) when Earth observers receive word from the traveler that she has arrived?

(iii) How much older will Earth observers calculate the traveler to be (measured from her frame) wen she reaches Vega than she was when she started the trip?

(10 marks)

SECTION B

Quantization

3. (a) Derive the formula for Compton scattering and give the final result in terms of the change in wavelength, $\Delta\lambda$. Fully explain all steps in your derivation.

(b) X-rays of wavelength $\lambda = 22$ pm are scattered from a carbon target, and the scattered rays are detected at 85° to the incident beam.

(i) What is the Compton shift of the scattered rays?

(ii) What percentage of the inital x-ray photon energy is transferred to an electron in such scattering?

(8 marks)

4. Light of wavelenth 200 nm shines on an aluminum surface; 4.20 eV is required to eject an electron. What is the kinetic energy of

(a) the fastest and

(b) the slowest ejected electron?

(c) What is the stopping potential for this situation?

(d) What is the cutoff wavelength for aluminum?

(6 marks)

SECTION C

The Schrödinger Equation

5. Consider an electron trapped in a potential well described by

$$U(x, y) = \begin{cases} 0, & \text{for } 0 \leq y \leq 2a \text{ and } 0 \leq x \leq 3a \\ \infty, & \text{otherwise} \end{cases}$$

(a) Solve the Schrödinger equation for the above potential.

(b) Show that the normalized wave function of the ground state (find the amplitude) is

$$\psi_{1,1}(x, y) = \sqrt{\frac{2}{3}}\frac{1}{a} \sin\left(\frac{\pi x}{3x}\right) \sin\left(\frac{\pi y}{2a}\right)$$

(c) Use the normalized ground state wave function to determine the expectation values of the x- and y-coordinates $\langle x \rangle$ and $\langle y \rangle$.

(d) Show that the energy can be written as

$$E = E_{n_x,n_y} = \frac{\hbar^2\pi^2}{2Ma^2}\left(\frac{n_x^2}{4} + \frac{n_y^2}{9}\right)$$

(12 marks)

6. Given that we can write the Laplacian in polar coordinates as

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2}.$$

Set up the differential equations that appear after separating the variables and solve the angular part of Schrödinger equation for the two dimensional central force problem. Comment on the boundary conditions and the effects thereof. (6 marks)

SECTION D

Atomic & Nuclear Physics

7. A helium atom is in an energy level with one electron occupying an s state and the other an f state. The two electron spins are antiparallel so that the spin magnetic moments cancel. The atom is placed in a magnetic field $B = 0.8$T.

(a) Sketch the resulting splitting of the original energy level

(b) What is the energy difference between adjacent levels of the resulting multiplet?

(4 marks)

8. State the Pauli exclusion principle (2 marks)

9. A 5 g charcoal sample from an ancient fire pit has a ^{14}C activity of 63 disintegrations/min. A living tree has a ^{14}C activity of 15.3 disintegrations/min per 1 g. The half-life of ^{14}C is 5730 years. How old is the charcoal sample? (4 marks)

10. (a) Why do isolated neutrons β^--decay as follows

$$n \to p + e^- + \overline{\nu}$$

(b) Taking into account the Pauli Principle, the Symmetry Effect and IPA draw a diagram and explain why large nuclei require more neutrons to be stable.

(6 marks)

-THE END-

Formula Sheet

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax) + C$$

$$\int x\sin^2(ax)dx = \frac{x^2}{4} - \frac{x}{4a}\sin(2ax) - \frac{1}{8a^2}\cos(2ax) + C$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E^2 = (pc)^2 + (mc^2)^2$$

$$\mu_B = \frac{e\hbar}{2m_e}$$

$$\Delta E = m\mu_B B$$

$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{mc}(1-\cos\theta)$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$\hbar c = 197 \text{ eV}\cdot\text{nm}$$

$$A_N = 6.022\times 10^{23}\text{mol}^{-1}$$

$$ke^2 = 1.44 \text{ eV}\cdot\text{nm}$$

$$E_R = 13.6 \text{ eV}$$

$$m_p = 938.3 \text{ MeV}/c^2$$

$$m_n = 939.6 \text{ MeV}/c^2$$

$$m_e = 0.511 \text{ MeV}/c^2$$